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Collective oscillations of a quasi-one-dimensional Bose condensate under damping

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Abstract

Affect of the damping on collective oscillations of a quasi-one-dimensional trapped repulsive Bose gas has been studied. Based on the phenomenological damping approach [L.P. Pitaevskii, Zh. Eksp. Teor. Fiz. 35 (1958) 408, Sov. Phys. JETP 35 (1958) 282] developed by Pitaevskii variational equations for the parameters of the condensate wave function have been derived. Analytical expressions for the condensate parameters in the steady-state have been obtained. Combined effect of the *resonant periodical variation of the trap strength* and *the damping* has been shown to change drastically asymptotical behavior of the driven norm oscillations. Bistability in nonlinear oscillations of the condensate under periodic variations of the trap potential is predicted.

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1. Introduction

The dynamics of a one-dimensional trapped ultra-cold Bose gas has attracted a great attention for last years [2,3]. Recently 1D regime has been realized experimentally in [4]. Measurements of the collective oscillations of such a system should give a lot of information about the BEC dynamics. In particular this is important in the analysis of the condensate dynamics in a magnetic waveguide, being a fundamental atom optical element [5].

A trapped 1D repulsive Bose gas is known [4] to be characterized by a single parameter $\zeta = mg_{1D}/(\hbar^2 n_{1D})$ which is the ratio between interaction energy and the kinetic energy of the ground state, m, g_{1D} and n_{1D} being atomic mass, the strength of interaction and 1D density correspondingly. Different regimes in one-dimensional geometry are possible depending on the density of gas. In the high density regime ($\zeta \ll 1$) the dynamics at low temperatures is described by a one-dimensional Gross–Pitaevskii equation with cubic mean field nonlinearity. The low density regime ($\zeta \gg 1$, Tonks–Girardeau (TG) regime) is characterized by the strong quantum correlations and a fermionic behavior of the system [6–8]. Modern experiments cover both of these limiting cases. Computations of the collective excitations frequencies of a trapped 1D repulsive Bose gas for different 1D configurations varying from the mean field regime to the TG regime were performed in [9]. In the present work we will concentrate our attention on the description of the quasi-one-dimensional dynamics of a repulsive BEC in the mean field regime.

Performed by now theoretical descriptions have mainly dealt with conservative systems (e.g., see [5,10]), where collective oscillations of a quasi-1D Bose–Einstein condensate (BEC) in the low and high density regimes were investigated. However the dissipation inheres in real systems. So it is of interest to investigate theoretically effect of damping on collective oscillations of a one-dimensional trapped repulsive Bose gas. The

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damping of the radial BEC oscillations in a cylindric trap, connected with the parametric resonance and leading to the energy transfer from collective oscillations to longitudinal sound waves has been studied in [11].

We consider here the problem using phenomenological approach developed by Pitaevskii [1] and employed later in [12]. Recently this approach has been successfully applied to a series of problems. For example, conditions for the parametric driving of dark solitons in repulsive quasi-one-dimensional BEC were found in [13], the analysis of the existence of stable 3D droplets in attractive BEC with nonlinearity management was carried out in [14], Faradey patterns in 2D BEC with damping were studied in [15] and so forth.

2. The model

To take into account the damping due to interaction of the condensate with the thermal cloud atoms we employ the phenomenological damping approach developed by Pitaevskii [1]. The dynamics of a trapped one-dimensional repulsive Bose gas with the damping in this case is described in the framework of the modified 1D Gross–Pitaevskii equation

$$i\hbar\phi_t = (1+i\gamma)\left(-\frac{\hbar^2}{2m}\phi_{xx} + V(x,t)\phi + g_{1\mathrm{D}}|\phi|^2\phi - \mu\phi\right) \tag{1}$$

with the total number of atoms $N = \int |\phi|^2\,dx$. The constant γ is the damping constant introduced phenomenologically to describe evolution toward equilibrium between the thermal cloud atoms and the condensate [12]. Approximate estimate obtained from the collision integral is given as $\gamma \sim 4Cm(akT)^2/(\pi\hbar^3)$, $C \approx 3$ [16]. As seen the dissipation constant can vary with the changes in temperature and atomic scattering length.

Eq. (1) is obtained for the case of a highly anisotropic external potential under the assumption that the transversal trapping potential is harmonic: $V(y,z) = m\omega_\perp^2(y^2+z^2)/2$ and $\omega_\perp \gg \omega_x$. Under such conditions we can consider the solution of 3D equation to have the form $U(x,y,z;t) = R(y,z)\phi(x,t)$ where $R_0^2 = m\omega_\perp \exp(-m\omega_\perp\rho^2/\hbar)/(\pi\hbar)$. Averaging the condensate wave function in radial direction we come to Eq. (1) describing the dynamics of the gas in longitudinal direction. The condition of 1D approximation is $\omega_\perp \gg \omega_x, \mu \ll \hbar\omega_\perp$, where μ is the chemical potential.

The potential $V(x,t)$ is assumed to be $V(x,t) = m\omega_x^2 x^2 \times F(t)$, where $F(t)$ describes the time dependence of the potential. The effective one-dimensional mean field nonlinearity coefficient $g_{1\mathrm{D}} = 2\hbar a_s\omega_\perp$, where a_s is the atomic scattering length. $a_s > 0$ corresponds to the Bose gas with a repulsive interaction between atoms and $a_s < 0$ to an attractive interaction. In this work we will study the case of repulsive condensate. Exact expression for this coefficient is given in [17] as $g_{1\mathrm{D}} = 2\hbar a_s\omega_\perp/(1+1.03a_s/l)$.

It is convenient to work with the dimensionless form of Eq. (1)

$$\begin{aligned} &i\psi_t + \frac{1}{2}\psi_{xx} - \frac{x^2}{2}F(t)\psi - g|\psi|^2\psi + \mu\psi \\ &\quad = -i\gamma\left(-\frac{1}{2}\psi_{xx} + \frac{x^2}{2}F(t)\psi + g|\psi|^2\psi - \mu\psi\right) \\ &\quad = R(\psi,\psi^*) \end{aligned} \tag{2}$$

by setting

$$t = \omega_x t, \qquad l = \sqrt{\hbar/(m\omega_x)},$$
$$x = x/l, \qquad \psi = \sqrt{2|a_s|\omega_\perp/\omega_x}\,\phi,$$

with $g = 1$ for the repulsive two-body interaction.

Any damping process eventually leads to an equilibrium state. Corresponding stationary solution of the GPE can be found from the equation

$$-\frac{1}{2}\psi_{xx} + \frac{x^2}{2}F\psi + g|\psi|^2\psi - \mu\psi = 0, \tag{3}$$

which, naturally, does not depend on the dissipative constant.

3. Variational analysis

For the wavefunction $\psi(x,t)$ we use the Gaussian trial function

$$\psi(x,t) = A(t)\exp\left(-\frac{(x-x_0(t))^2}{2a^2(t)} + \frac{ib(t)(x-x_0(t))^2}{2} + i\kappa(t)\big(x-x_0(t)\big) - i\varphi(t)\right), \tag{4}$$

where A, a, b, x_0, κ and φ are the amplitude, width, chirp, position, momentum and linear phase, respectively.

The choice of the ansatz in the Gaussian form is motivated by both convenience and the fact, that in a harmonic trap potential such a choice is a good approximation for the condensate wave function. Comparison of different ansatzes in variational analysis of BEC is given in [18]. Moreover, as it will be seen later, the numerical simulations performed in this work confirm this approximation.

Eq. (2) can be obtained from the variational equations [19–21]

$$\frac{\partial L}{\partial\psi^*} - \frac{\partial}{\partial x}\frac{\partial L}{\partial\psi_x^*} - \frac{\partial}{\partial t}\frac{\partial L}{\partial\psi_t^*} + \frac{\delta L_R}{\delta\psi^*} = 0, \tag{5}$$

where L is the Lagrangian density, $L \equiv L(x,t)$, of a conservative system, given by

$$L = \frac{i}{2}\big(\psi_t\psi^* - \psi_t^*\psi\big) - \frac{1}{2}|\psi_x|^2 - \left(\frac{x^2}{2}F(t) - \mu\right)|\psi|^2 - \frac{g}{2}|\psi|^4 \tag{6}$$

and L_R is defined as $\partial L_R/\partial\psi^* = -R(\psi,\psi^*)$, where $R(\psi,\psi^*)$ is the right side of Eq. (2).

Inserting trial function (4) into Eq. (6) and averaging it as

$$\bar{L} = \int L(x,t)\,dx \tag{7}$$

we obtain the averaged Lagrangian of the conservative system in terms of the trial function parameters:

$$\frac{\bar{L}}{\sqrt{\pi}} = -A^2 a\left(\frac{a^2 b_t}{4} - \varphi_t - \kappa x_{0t} + \frac{1}{4a^2} + \frac{a^2b^2}{4} + \frac{1}{2}\kappa^2 + \frac{(a^2+x_0^2)F}{4} + \frac{gA^2}{2\sqrt{2}} - \mu\right). \tag{8}$$

Using Eq. (5) and its conjugate, we obtain a system of equations for the variational parameters η_i [20,21]:

$$\frac{\partial \bar{L}}{\partial \eta_i} - \frac{d}{dt}\frac{\partial \bar{L}}{\partial \eta_{it}} = \int dx\left(R\frac{\partial \psi^*}{\partial \eta_i} + R^*\frac{\partial \psi}{\partial \eta_i}\right). \tag{9}$$

Inserting Eqs. (4) and (8) into Eq. (9) one can derive the following system of ordinary differential equations (ODE) for the condensate parameters:

$$\begin{aligned}
\frac{db}{dt} &= \frac{1}{a^4} - b^2 - F + \frac{gA^2}{\sqrt{2}a^2} - \frac{2\gamma b}{a^2},\\
\frac{da}{dt} &= ab + \gamma\left(\frac{1}{2a} - \frac{a^3b^2}{2} - \frac{a^3F}{2} + \frac{gA^2a}{2\sqrt{2}}\right),\\
\frac{d(A^2a)}{dt} &= -A^2a\gamma\left(\frac{a^2b^2}{2} + \frac{1}{2a^2} + \kappa^2 + \frac{(a^2+2x_0^2)F}{2} + \frac{\sqrt{2}gA^2a}{a} - 2\mu\right),\\
\frac{d\varphi}{dt} &= \frac{1}{2a^2} + \frac{5gA^2a}{4\sqrt{2}a} + \frac{\kappa^2}{2} - \kappa x_{0t} - \mu - \frac{\gamma b}{2},\\
\frac{d\kappa}{dt} &= -x_0F - \gamma\left(\left(\frac{1}{a^2} + a^2b^2\right)\kappa + a^2bx_0F\right),\\
\frac{dx_0}{dt} &= \kappa - \gamma a^2(b\kappa + Fx_0).
\end{aligned} \tag{10}$$

This system can be also derived using the method of moments [19,21].

Introducing new parameter, the norm $N = A^2a\sqrt{\pi}$ instead of the amplitude A, one can divide set of differential equations (10) into three groups. A set of differential equations for parameters b, a, N is

$$\begin{aligned}
\frac{db}{dt} &= \frac{1}{a^4} - b^2 - F + \frac{gN}{\sqrt{2\pi}a^3} - \frac{2\gamma b}{a^2},\\
\frac{da}{dt} &= ab + \gamma\left(\frac{1}{2a} - \frac{a^3b^2}{2} - \frac{Fa^3}{2} + \frac{gN}{2\sqrt{2\pi}}\right),\\
\frac{dN}{dt} &= -\gamma N\left(\frac{a^2b^2}{2} + \frac{1}{2a^2} + \kappa^2 + \frac{(a^2+x_0^2)F}{2} + \frac{2gN}{\sqrt{2\pi}a} - 2\mu\right).
\end{aligned} \tag{11}$$

A set for parameters κ, x_0 is

$$\begin{aligned}
\frac{d\kappa}{dt} &= -x_0F - \gamma\left[\left(\frac{1}{a^2} + a^2b^2\right)\kappa + a^2bx_0F\right],\\
\frac{dx_0}{dt} &= \kappa - \gamma a^2(b\kappa + Fx_0).
\end{aligned} \tag{12}$$

And an equation for the phase ϕ is

$$\frac{d\varphi}{dt} = \frac{1}{2a^2} + \frac{5gN}{4\sqrt{2\pi}a} + \frac{\kappa^2}{2} + \frac{x_0^2F}{4} - \kappa x_{0t} - \mu + \frac{\gamma b}{2}. \tag{13}$$

Taking into consideration that in an equilibrium state derivatives $b_t = 0$, $a_t = 0$, $N_t = 0$, $\kappa_t = 0$ and $x_{0t} = 0$ one can obtain the following stationary solutions of Eqs. (11)–(12):

$$\begin{aligned}
&b_s = 0, \qquad a_s = \sqrt{\frac{4\mu + \sqrt{16\mu^2 + 60F}}{10F}},\\
&N_s = \frac{8\sqrt{\pi}}{5g}\left(\mu a_s - \frac{1}{2a_s}\right),\\
&\kappa_s = 0, \qquad x_{0s} = 0.
\end{aligned} \tag{14}$$

Let us separately consider set of two differential equations (12) for κ and x_0. Differentiating the last equation for x_0 by time and making corresponding substitutions we reduce the two equations to one

$$\frac{d^2x_0}{dt^2} + \gamma\left(a^2F + \frac{1}{2a^2} + \frac{a^2b^2}{2}\right)\frac{dx_0}{dt} + \frac{x_0F}{2} = 0. \tag{15}$$

As seen the solutions of this equation are strongly damped ones and the stationary solution $x_{0s} = 0$ is stable. The same is valid for the parameter κ. Moreover here only *parametric resonance* may be realized. Taking it into account, one can consider the set (11) with $\kappa(t) = \kappa_s = 0$ and $x_0(t) = x_{0s} = 0$ to describe resonance phenomena.

Applying the procedure used in obtaining Eq. (12) one can reduce the set of three differential equations (11) to a set of two ones:

$$\begin{aligned}
a_{tt} &= \frac{1}{a^3} - aF + \frac{gN}{\sqrt{2\pi}a^2} - \gamma\left(\frac{3}{a^2} + a_t^2 + a^2F + \frac{gN}{2\sqrt{2\pi}a^2}\right)a_t + \frac{\gamma a^3F_t}{2}\\
&= 0,\\
N_t &= -\gamma N\left(\frac{1}{2a^2} + \frac{a_t^2}{2} + \frac{a^2F}{2} + \frac{2gN}{\sqrt{2\pi}a^2} - 2\mu\right).
\end{aligned} \tag{16}$$

It should be noted that here the trap strength F may depend on time.

3.1. Resonant suppression of the norm

Let us consider a periodical variation of the parabolic trap strength in time

$$F(t) = F_0(1 + h\sin\omega t), \tag{17}$$

where F_0 is constant part of the trap strength F, h is the relative amplitude of oscillations which is supposed to be small, ω is the frequency of driven oscillations. To describe evolution of the width a and norm N under periodical variation of the trap strength we expand them near the stationary points $a(t) = a_s + a_1(t)$, $N(t) = N_s + N_1(t)$. Corresponding stationary values a_s and N_s are determined by expressions (14). Substituting the expansion for a into the first equation of Eq. (16) and holding the terms of the first order of a_1, h and γ we come to the following equation

$$a_{1tt} + \lambda(N)a_{1t} + \omega_0^2 a_1 = -ha_s F_0 \sin\omega t, \tag{18}$$

where

$$\omega_0^2 = \frac{1}{a_s^4} + 3F_0,$$

$$\lambda(N_s) = \gamma\left(\frac{3}{a_s^2} + a_s^2 F_0 + \frac{gN_s}{2\sqrt{2\pi}a_s}\right) \tag{19}$$

are the eigenfrequency and effective damping coefficient correspondingly. It should be noted that $\lambda > 0$ for $\gamma > 0$.

As readily seen this equation describes main resonance in the width oscillations if $\omega \approx \omega_0$. At $\omega = \omega_0$ the amplitude of the width oscillations is maximal and determined by formula [22]

$$a_{1\max} = \frac{ha_s F_0}{\lambda(N_s)\omega_0}. \tag{20}$$

Substituting obtained resonant solution for a into the second equation of set (16) and averaging it over the period $T = 2\pi/\omega$, for the steady-state norm we get

$$\tilde{N}_s = N_s - \frac{\sqrt{2\pi}a_s}{2g}\left(\mu + \frac{1}{2a_s^2} + \frac{a_s^2\omega_0^2}{4}\right)\left(\frac{hF_0}{\lambda(N_s)\omega_0}\right)^2. \tag{21}$$

One can see that *resonant* variation of the trap strength causes *decreasing* in the steady-state value of the norm N which is in inverse proportion to *the damping* γ.

4. Numerical simulations

We have carried out a series of time dependent simulations of the system based on the variational approach using Eq. (16) as well as exact numerical calculations using Eq. (2). In our numerical calculations we discretize the problem in a standard way, with the time step dt, and spatial step dx, so ψ_j^k approximates $\psi(j\,dx, k\,dt)$. More specifically we approximate the governing Eq. (2) with the semi-implicit Crank–Nickolson scheme using split-step method [23]. The results of numerical simulations of both PDE and ODE models are presented below.

Fig. 1 shows time evolution of the width and the norm of the condensate with $\mu = 2$ for the case of $h = 0$, i.e., when the external potential *has no time perturbations*. Initial excitation of the BEC width oscillations for this case is prepared in the following way. Firstly initial wave packet is chosen to be in the equilibrium state defined by Eqs. (3) and (14) and corresponding to the trap strength $F \equiv 0.9$. The PDE and ODE simulations start with this initial wave packet and then the value of $F(t)$ is gradually increased from 0.9 to 1 with a transition time of $\Delta t = 2$. Beginning from the time $t = 2$ the evolution goes on at constant value of $F = 1$.

We see that the ODE simulation leads to an equilibrium state the norm of which is 3–4 percent less than that of the PDE results.

Fig. 2 presents the norm behavior when *the trap strength is periodically varied in time* as Eq. (17). An interesting behavior of the norm is observed here. If an external periodical perturbation of the resonant frequency is applied to a trapped BEC which is already in the equilibrium state then the norm of the condensate starts to decrease going to new steady state. In the figure the frequency ω of the periodical trap perturbation is taken to be equal to the eigenfrequency ω_0 of the system determined from Eq. (19). Numerical simulations have been carried

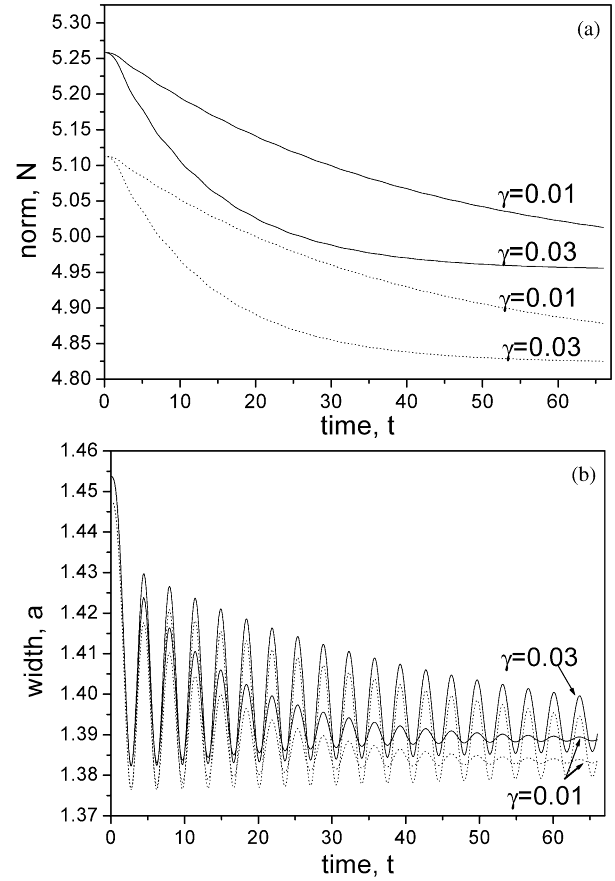

Fig. 1. Dynamics of the norm (a) and the width (b) of the repulsive BEC in a harmonic trap without perturbation. Solid and dotted lines show PDE and ODE results.

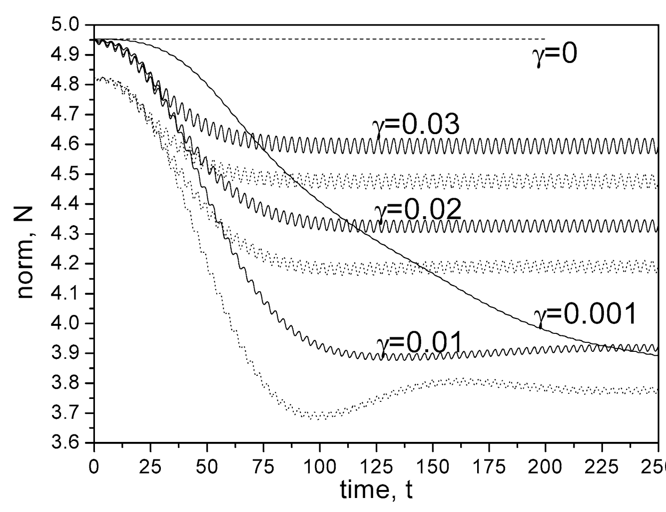

Fig. 2. Behavior of the norm of a trapped BEC when the trap starts oscillating with the amplitude $h = 0.06$. Initially the BEC is in the equilibrium state. The solid lines stand for full numerical simulations of the PDE, while the dotted lines represent the ODE results.

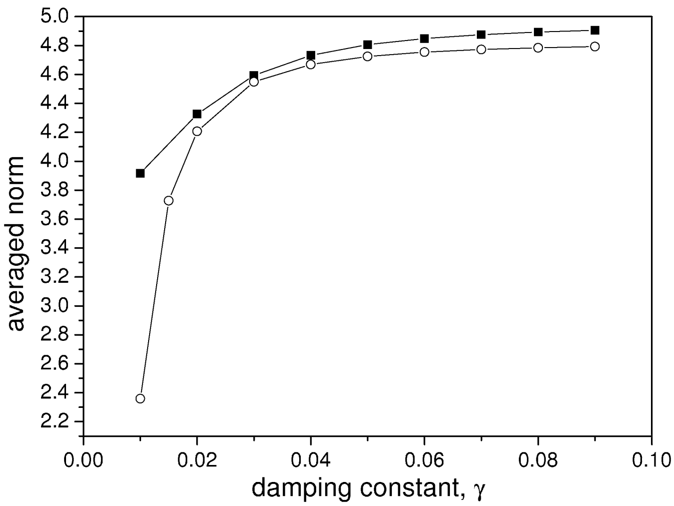

Fig. 3. Averaged steady state values of the BEC norm versus the damping constant γ in the main resonance for the case $h = 0.06$, $\mu = 2$. Squares are for PDE calculations, circles are for theoretical values obtained from formula (21).

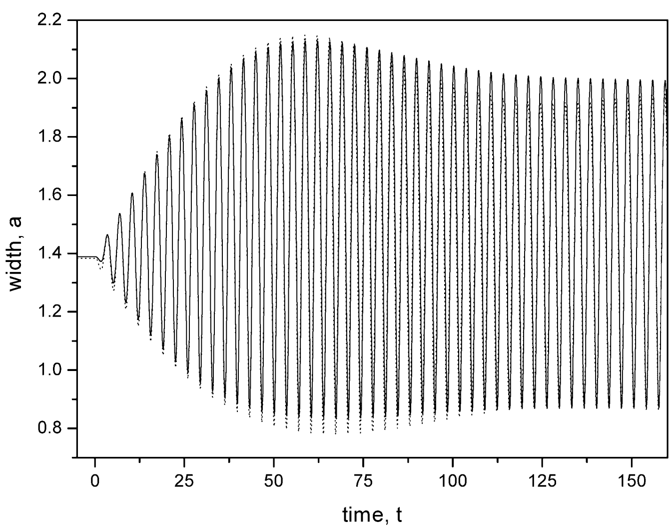

Fig. 4. The BEC width versus time t in the main resonance for the case $h = 0.06$, $\mu = 2$, $\gamma = 0.01$. Solid and dotted lines show the PDE and ODE results, respectively.

out for different values of γ at the same amplitude of the trap oscillations, $h = 0.06$. One can see that combined effect of *the damping* and *resonant periodical variation of the trap strength* causes suppression of the averaged steady state value of the driven norm, smaller values of the damping constant γ leading to more strong suppression of the norm. The effect is explained by that at smaller values of γ the amplitude of the width oscillations becomes greater and in accordance with the second equation of set (16) averaged value of the steady state norm decreases.

Comparison of the results of theoretical prediction, Eq. (21) and full GPE calculations of averaged steady state values of the norm for different values of the damping γ is shown in Fig. 3. Relative amplitude of the trap strength h oscillations in all calculations is chosen to be 0.06. One can see that theoretical description of resonant suppression of the norm steady state

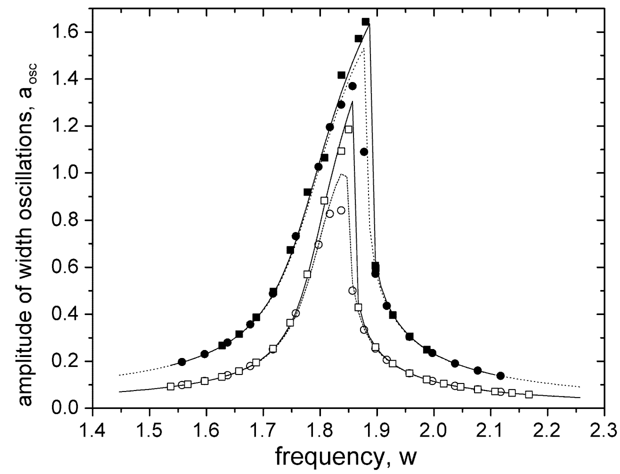

Fig. 5. Oscillation amplitude as predicted by the PDE (scatter) and ODE (line) models. Here $\gamma = 0.005$ (solid lines and squares) and $\gamma = 0.01$ (dotted lines and circles). The upper lines are for the case $h = 0.06$, the lower lines are for the case $h = 0.03$. The chemical potential $\mu = 2$.

is in a good agreement with full PDE simulations. Discrepancy for the values of damping constant $\gamma < 0.015$ is due to that for small γ *the resonance* we describe becomes nonlinear, whereas the theoretical description, Eq. (21) is obtained under supposition of *linear resonance*.

The width dynamics under main resonance with the initial wave packet taken in the equilibrium state is depicted in Fig. 4. As shown, in contrast to the norm the width oscillates near the previous point. As calculations show, the amplitude of the width oscillations becomes stable by the time $t = 140$.

Performing ODE and PDE simulations for a number of frequencies in the vicinity of the resonant frequency of BEC oscillations and measuring amplitudes of steady oscillations, we have plotted the values of the oscillation amplitude as a function of the frequency of the periodical trap perturbations with $h = 0.03$ and $h = 0.06$ for the cases $\gamma = 0.01$ and $\gamma = 0.005$ (see Fig. 5).

We see that, e.g., when $h = 0.06$ and $\gamma = 0.005$ the highest amplitude of oscillations is driven in the trap perturbation with the frequency $\omega = 1.89$ which is more than the eigenfrequency $\omega_0 = 1.809$. Bistability appears with the smaller values of γ in the vicinity of this critical frequency. For $\omega = 1.88$ we observe large oscillations with $a_{osc} = 1.6$, while at $\omega = 1.91$ we observe much smaller width oscillations with $a_{osc} = 0.6$. It can be seen that with the growth of trap perturbations the value of the critical frequency becomes greater.

Estimates for the experiment parameters are the following. The magnetic trap can be taken with parameters $\omega_\perp = 2\pi \times 400$ Hz, $\omega_x = 2\pi \times 10$ Hz, and the number of atoms of ^{7}Li $N = 0.19 \times 10^4$ with a scattering length $a_s = 0.4$ nm (repulsive gas). Then, the eigenfrequency of the harmonically trapped BEC $\omega_0 = 1.809 \times \omega_x = 2\pi \times 18.09$ Hz. By applying the external perturbation $F(t) = 1 + 0.06\sin(\omega t)$ with $\omega = 2\pi \times 18.09$ Hz to the trapped BEC one will observe decreasing of the number of atoms by 20, 13 and 7 percent for $\gamma = 0.1$, 0.2 and 0.3, respectively. For the external perturbation with $\omega = 2\pi \times 18.79$ Hz, large oscillations will be observed with $a_{\text{osc}} = 1.6l = 1.6\sqrt{\hbar/(m\omega_x)} = 5.5$ µm, while at $\omega = 2\pi \times 19.11$ Hz one will observe much smaller oscillations with $a_{\text{osc}} = 0.6l = 2.06$ µm.

5. Conclusions

In this Letter we have studied collective oscillations of a quasi-one-dimensional Bose gas in the presence of dissipative effects. The modified Gross–Pitaevskii equation in the framework of the phenomenological approach [1,12] has been employed. To describe evolution of oscillations we use the modified variational approach taking into account the dissipation. The results obtained from computation of the system of equations for the wave function parameters are confirmed direct numerical simulations of the full GP equation.

The expressions for the width and the norm of a condensate in an equilibrium state have been derived analytically.

Main resonance in the condensate oscillations has been studied. From consideration of the variational equation (10) we found that periodical resonant modulation of the trap potential in the modified GP equation (1) drastically change asymptotical behavior of the norm. As known a ground state of the governing equation (1) does not depend on the damping constant γ *by definition* and corresponding asymptotical value of the solution norm depends only on the chemical potential μ. In the case of time dependent modulation of the trap we have shown the combined effect of *resonant periodical modulation of the trap strength* and *the damping* to change drastically asymptotical behavior of the driven norm oscillations and cause suppression of the averaged steady state value of the driven norm, smaller values of the damping constant γ leading to more strong suppression of the norm.

Unlike the norm oscillations, the condensate width remains to oscillate about the same mean value independent on the damping constant γ.

We have also shown that in resonances the bistability appears with smaller values of the dissipative constant in the vicinity of the critical frequency which is above the eigenfrequency of the BEC.

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